Cournot 2 firms and linear demand

Problem Statement

In this exercise, we are to solve a Cournot duopoly with linear demand and different constant marginal costs. The market demand function is given by $P = a - bQ$. The firms have constant marginal costs, denoted by c_1 and c_2. Find the equilibrium quantities and price for both firms.

Introduction

Consider a Cournot duopoly where two firms compete in quantity in a market with linear demand. The market demand function is given by:

$$P = a - bQ, \tag{1}$$

where P is the price, $Q = q_1 + q_2$ is the total quantity in the market, q_1 and q_2 are the quantities produced by firm 1 and 2, respectively, and a and b are positive constants. Both firms have different but constant marginal costs, denoted by c_1 and c_2.

Solution

Profit function

The profit function for each firm is defined as revenue minus cost, as a function of the quantity produced. For firm 1:

$$\pi_1 = Pq_1 - C_1(q_1) = (a - bQ)q_1 - c_1 q_1 = (a - b(q_1 + q_2) - c_1)q_1. \tag{2}$$

Similarly, for firm 2:

$$\pi_2 = (a - b(q_1 + q_2) - c_2)q_2. \tag{3}$$

Profit maximization

Each firm maximizes its profits by choosing its production quantity, taking the quantity produced by the other firm as given. Maximization is performed by differentiating the profit function with respect to the quantity produced and solving for each firm.

First, maximize π_1 with respect to q_1:

$$\frac{\partial \pi_1}{\partial q_1} = a - 2bq_1 - bq_2 - c_1 = 0. \tag{4}$$

From this, we obtain the reaction function for firm 1:

$$q_1 = \frac{a - c_1}{2b} - \frac{1}{2}q_2. \tag{5}$$

Similarly, maximize π_2 with respect to q_2:

$$\frac{\partial \pi_2}{\partial q_2} = a - bq_1 - 2bq_2 - c_2 = 0. \tag{6}$$

From this, we obtain the reaction function for firm 2:

$$q_2 = \frac{a - c_2}{2b} - \frac{1}{2}q_1. \tag{7}$$

Cournot equilibrium

To find the Cournot equilibrium, we solve the system of equations

$$\begin{cases} q_1 = \frac{a-c_1}{2b} - \frac{1}{2}q_2 \\ q_2 = \frac{a-c_2}{2b} - \frac{1}{2}q_1 \end{cases}. \tag{8}$$

To solve this system of equations, we can substitute q_2 from the second equation into the first equation, which gives us:

$$q_1 = \frac{a - c_1}{2b} - \frac{1}{2}\left(\frac{a - c_2}{2b} - \frac{1}{2}q_1\right). \tag{9}$$

Solving this equation for q_1, we get:

$$q_1 = \frac{2a - 2c_1 + c_2}{3b}. \tag{10}$$

Now, we substitute the value of q_1 into the equation for q_2:

$$q_2 = \frac{a - c_2}{2b} - \frac{1}{2}\left(\frac{2a - 2c_1 + c_2}{3b}\right). \tag{11}$$

Solving this equation for q_2, we get:

$$q_2 = \frac{2a - 2c_2 + c_1}{3b}. \tag{12}$$

Finally, we calculate the equilibrium price using the market demand function and the values of q_1 and q_2:

$$P = a - b(q_1 + q_2) = a - b\left(\frac{2a - 2c_1 + c_2}{3b} + \frac{2a - 2c_2 + c_1}{3b}\right). \tag{13}$$

Simplifying, we get:

$$P = \frac{a + c_1 + c_2}{3}. \tag{14}$$

Thus, the equilibrium quantities for firms 1 and 2 are $q_1 = \frac{2a-2c_1+c_2}{3b}$ and $q_2 = \frac{2a-2c_2+c_1}{3b}$, respectively, and the equilibrium price is $P = \frac{a+c_1+c_2}{3}$.

The profit for firm 1 at the Cournot equilibrium is:

$$\pi_1^* = (P - c_1)q_1 \tag{15}$$

$$= \left(\frac{a + c_1 + c_2}{3} - c_1\right)\left(\frac{2a - 2c_1 + c_2}{3b}\right). \tag{16}$$

The profit for firm 2 at the Cournot equilibrium is:

$$\pi_2^* = (P - c_2)q_2 \tag{17}$$

$$= \left(\frac{a + c_1 + c_2}{3} - c_2\right)\left(\frac{2a - 2c_2 + c_1}{3b}\right). \tag{18}$$